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TCOM 151

Slab Waveguides

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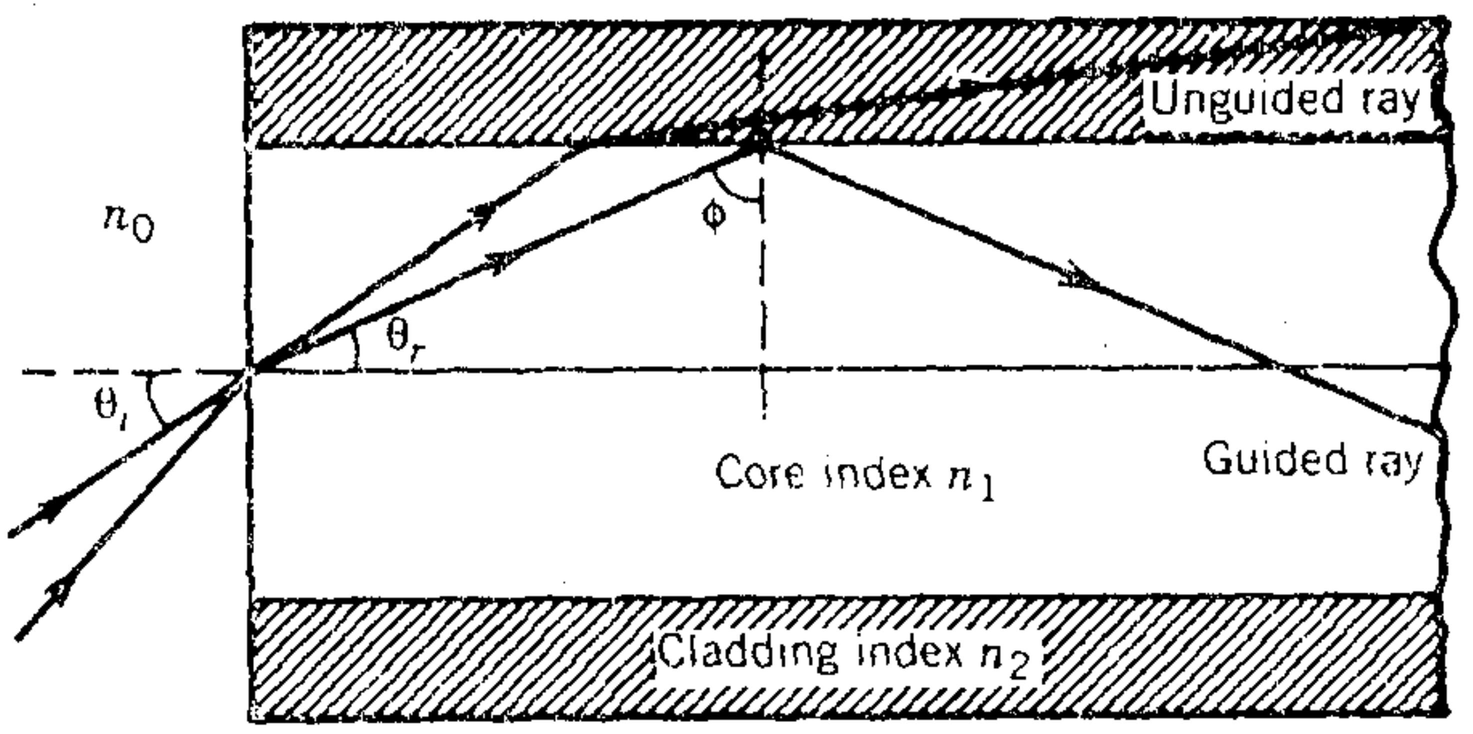
Primary Reference: **C. Pollock, *Fundamentals of Optoelectronics, Chapter 2, Richard Irwin Inc, (1995), ISBN 0-256-10104-3***

Numerical Aperture: Ray Optics



What is the maximum incident angle θ_1 that will enable waveguiding of the optical ray?

Numerical Aperture

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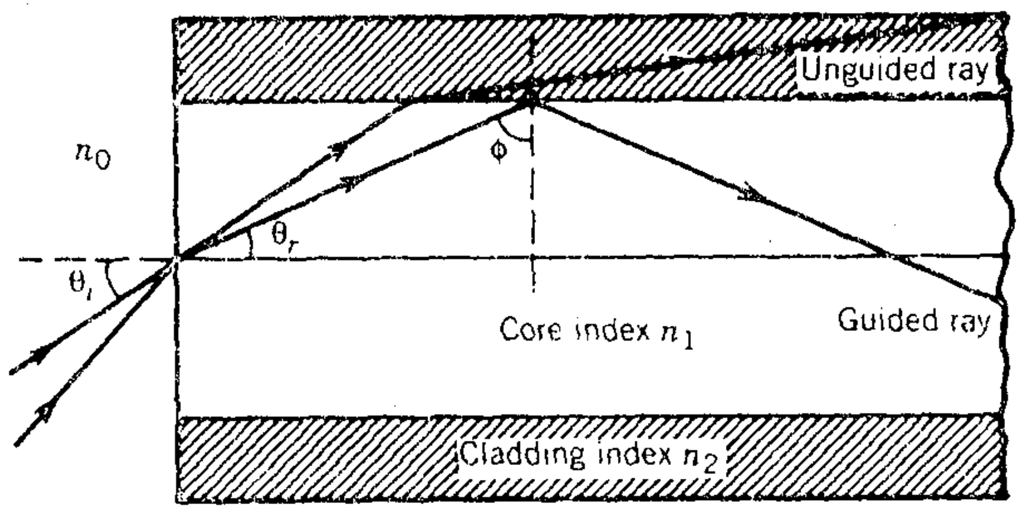

- $\theta_{1,max}$ is a function of $\phi_{1,min}$. Why?
- $\phi_{1,min}$ is actually ϕ_{TIR} !
- $n_1 \sin(\phi_{TIR}) = n_2 \sin(\pi/2)$
- For coupling into the waveguide,
 $\sin\theta_i$ has to be less than $n_1\sin\theta_r$ (via Snell's Law)
- Therefore $\sin\theta_i < \sin\theta_{i,max} = n_1\sin(\pi/2 - \theta_{TIR})$
- Numerical Aperture = N.A. $\equiv \sin\theta_{1,max}$

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Numerical Aperture = $(n_1^2 - n_2^2)^{1/2}$

- Numerical Aperture = N.A. $\equiv \sin\theta_{i,max} = n_1 \sin(\pi/2 - \phi_{TIR}) = n_1 \cos \phi_{TIR}$
- $n_1 \sin(\phi_{TIR}) = n_2 \sin(\pi/2) = n_2$
- $[n_1 \sin(\phi_{TIR})]^2 = [n_2]^2 \rightarrow \sin^2(\phi_{TIR}) = [n_2/n_1]^2$
- $\sin^2(\phi_{TIR}) + \cos^2(\phi_{TIR}) = 1 \rightarrow [n_2/n_1]^2 + \cos^2(\phi_{TIR}) = 1$

Now we are ready to develop an expression for NA ...

N.A. $= n_1\cos(\theta_{TIR}) = n_1 (1 - [n_2/n_1]^2)^{1/2} = (n_1^2 - n_2^2)^{1/2}$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Let's assume the case when $n_1 \approx n_2$ s.t. $\Delta << 1$

$(n_1 - n_2)^{1/2} (n_1 + n_2)^{1/2} = (\Delta\, n_1\, 2\, n_1)^{1/2} = n_1 (2\Delta)^{1/2}$

Hence **N.A.** $\approx n_1 (2\Delta)^{1/2}$

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Examples

n_1	n_2	Δ	NA	$\theta_i = \theta_{NA}$
1.446	1.440	4.149×10^{-3}	0.1316	7.56^o
1.446	1.389	0.0394	0.40	23.6^o
1.446	1.000	0.3084	1.044	?????

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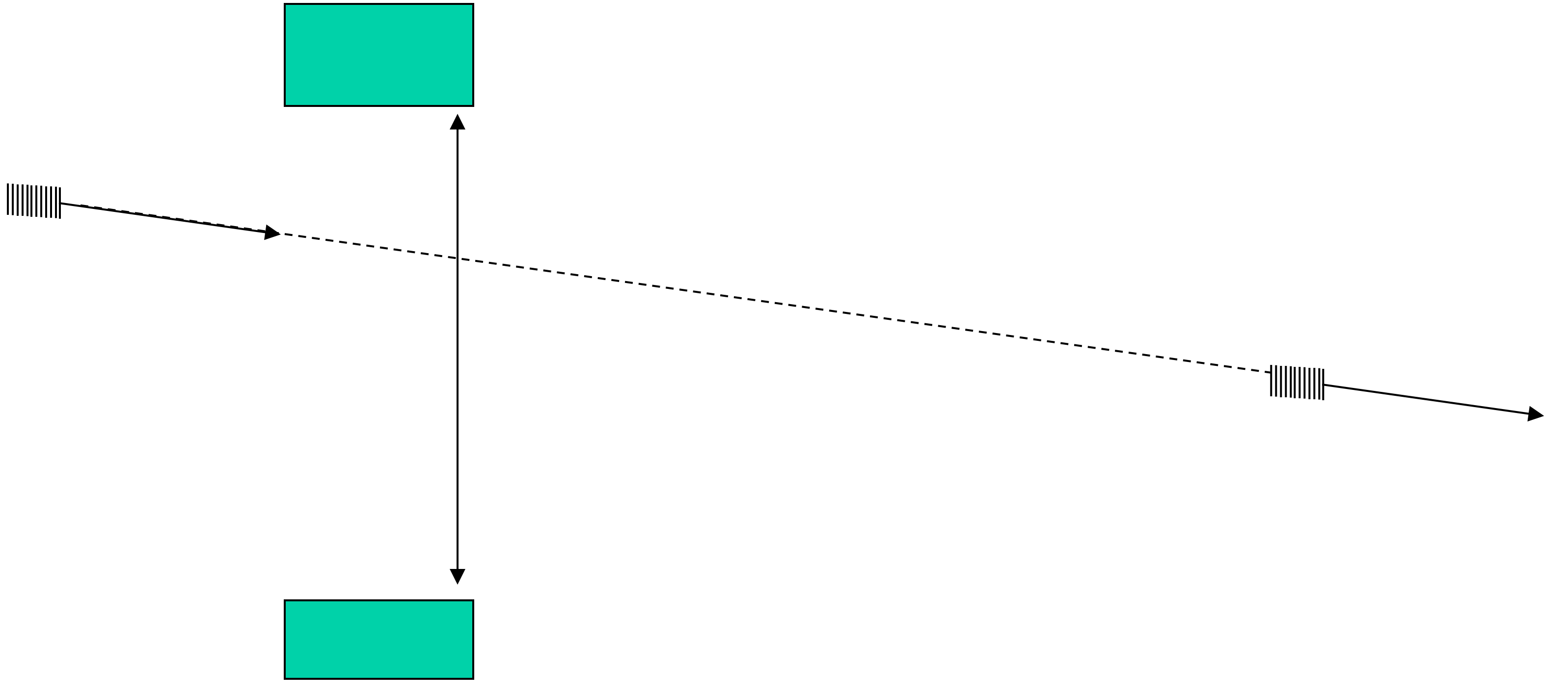
From Rays to Waves

- Ray picture is inadequate to explain all observed phenomena in light propagation in waveguides
- The wave picture becomes significant when the waveguide dimensions approach the wavelength of light



From Rays to Waves

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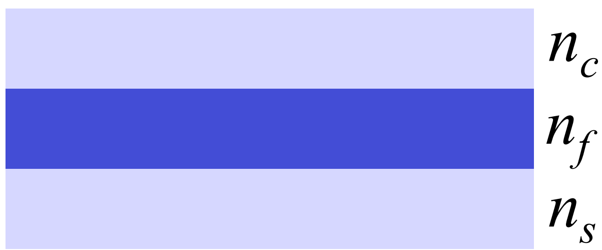
Simplest Waveguide

- Let’s look at the planar slab waveguide
- Step-index/confinement in the x-direction
- Infinite in extent in the z and y directions



- Solve the wave equation with B.C.s at the interfaces
- This will lead to the concept of modes

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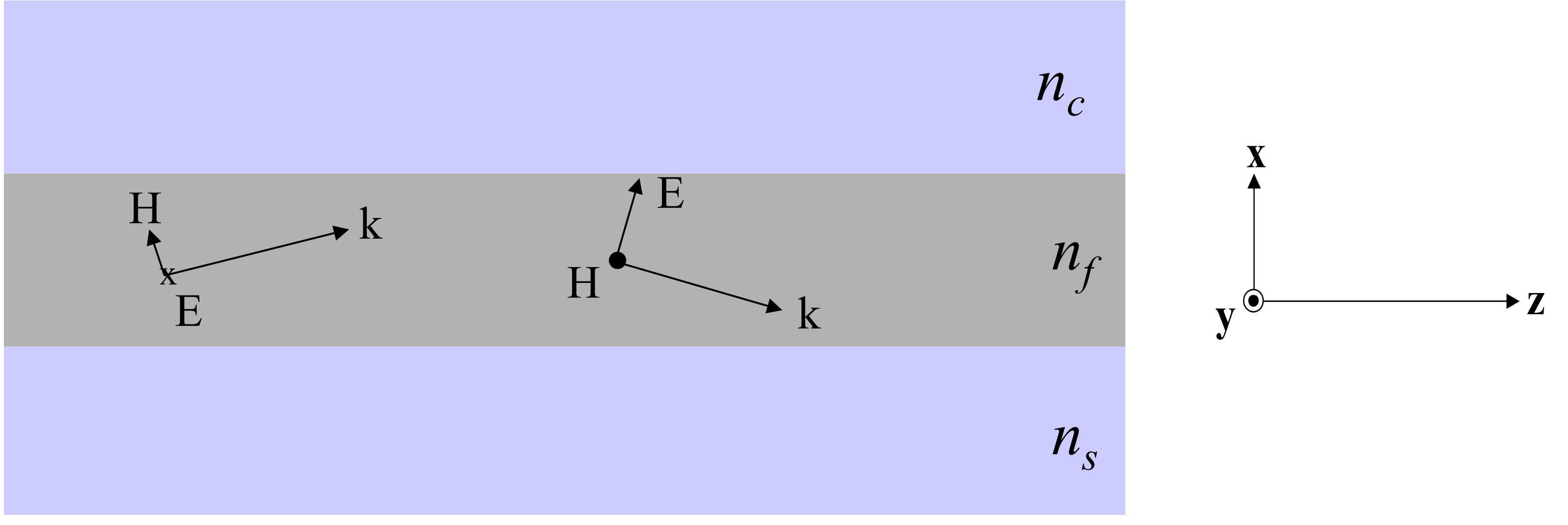
TE and TM Waves



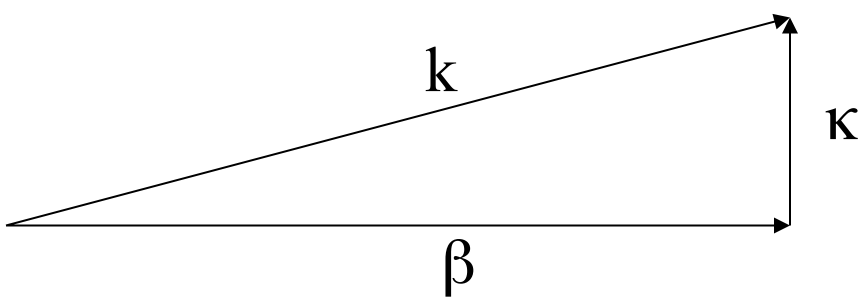
- +z is the direction of propagation
- Transverse Electric - no longitudinal E
- Transverse Magnetic - no longitudinal H
- $n_f > n_s > n_c$
- TE case: $\nabla^2 E_y + k_o{}^2 n_i{}^2 E_y = 0$, n_i = function of layer, x, only

Solution

- TE case: $\nabla^2 E_y + k_o^2 n_i^2 E_y = 0$, n_i = function of layer, x, only
- $E_y(x,z) = E_y(x) \exp(-j\beta_i z)$, trial solution, beta is prop. coeff.
- Plugging into the wave equation, with $E_y'' = 0$ we get:

  $$\nabla_{xx} E_y + (k_o^2 n_i^2 - \beta^2) E_y = 0, \; n_i \text{ is a function of layer}$$

- Decaying solution $k_o n_i < \beta$; $E_y(x) = E_o e^{-\gamma x}$; attenuation coeff.
- Oscillatory solution $k_o n_i > \beta$; $E_y(x) = E_o e^{\pm j\kappa x}$; transverse wavevector
- Total wavevector k is the vector sum of the transverse wavevector and the propagation constant (longitudinal component)



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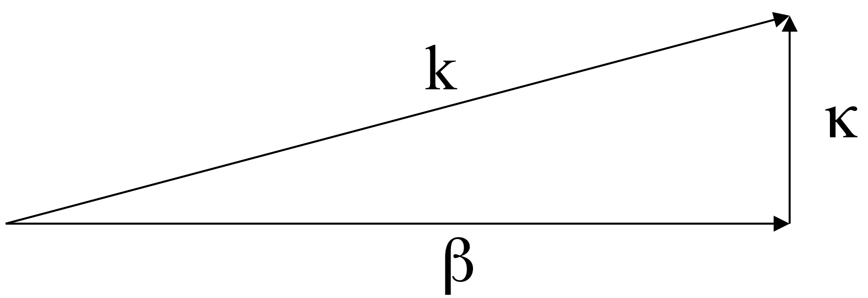
Wavevectors: $\nabla_{xx}E_y + (k_o^2 n_i^2 - \beta^2)\, E_y = 0$

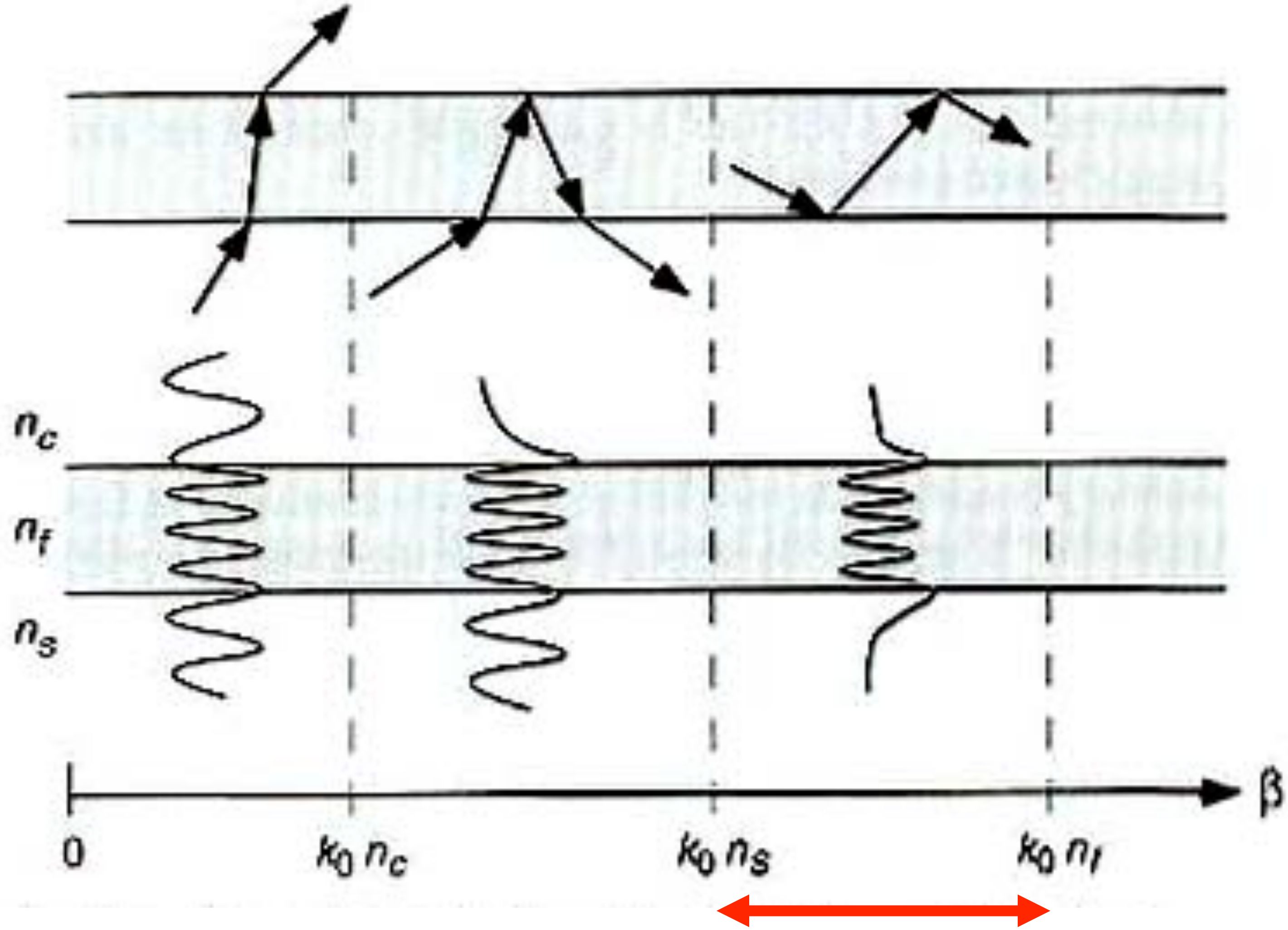
$$E_o\, e^{\pm j\kappa x} \Longrightarrow \quad \kappa = (k_o^{\,2} n_i^{\,2} - \beta^{\,2})^{1/2}$$

$$E_o\, e^{-\gamma x} \Longrightarrow \quad \gamma = (\beta^{\,2} - k_o^{\,2} n_i^{\,2})^{1/2}$$



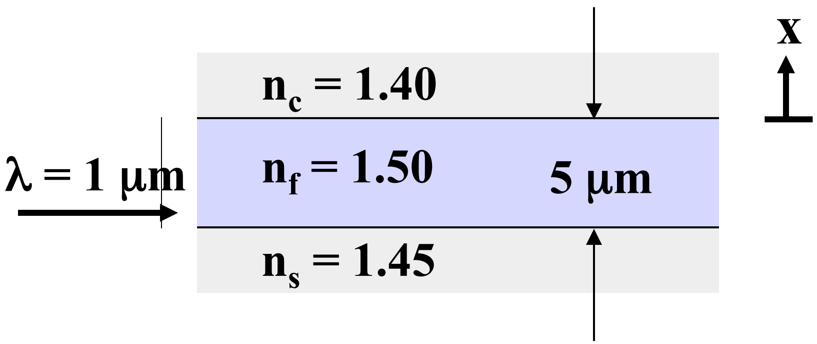
- wavevector
- transverse wavevector
- propagation constant

Slab Waveguide: β Characterizes Modes



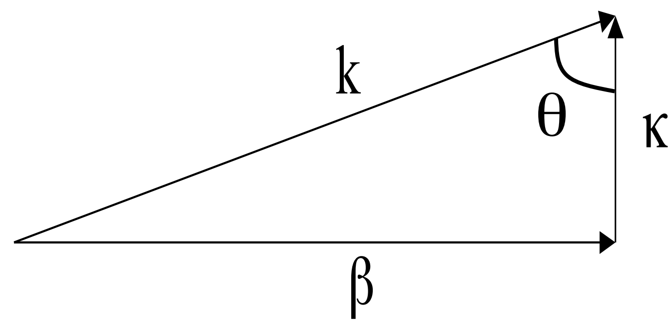
Varying the angle between k and z from 90 deg to 0 deg varies β

Solving the TE Characteristic Eqn.



- $\gamma_c = \gamma_c(\kappa_f) = (\beta^2 - k_o^2\ 1.4^2)^{1/2}$
- $\gamma_s = \gamma_s(\kappa_f) = (\beta^2 - k_o^2\ 1.45^2)^{1/2}$
- $\beta = \beta(\kappa_f) = (k_o^2\ 1.5^2 - \kappa_f^2)^{1/2}$

- The minimum value of κ_f is 0, when the wavevector $k = k_o n_f = \beta$
- The maximum value of κ_f is $(k_o^2\ n_f^2 - k_o^2\ n_s^2)^{1/2}$



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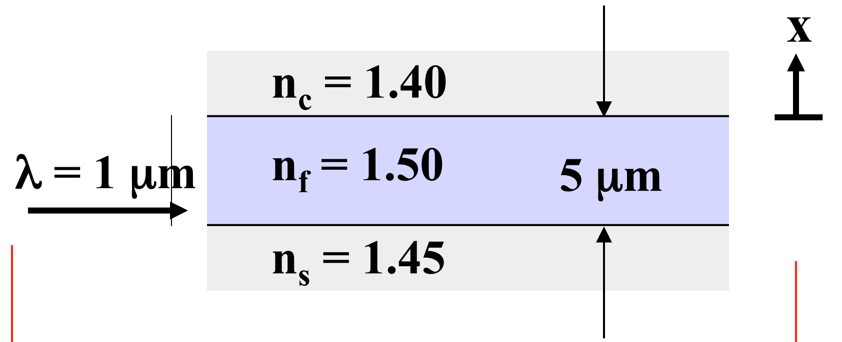
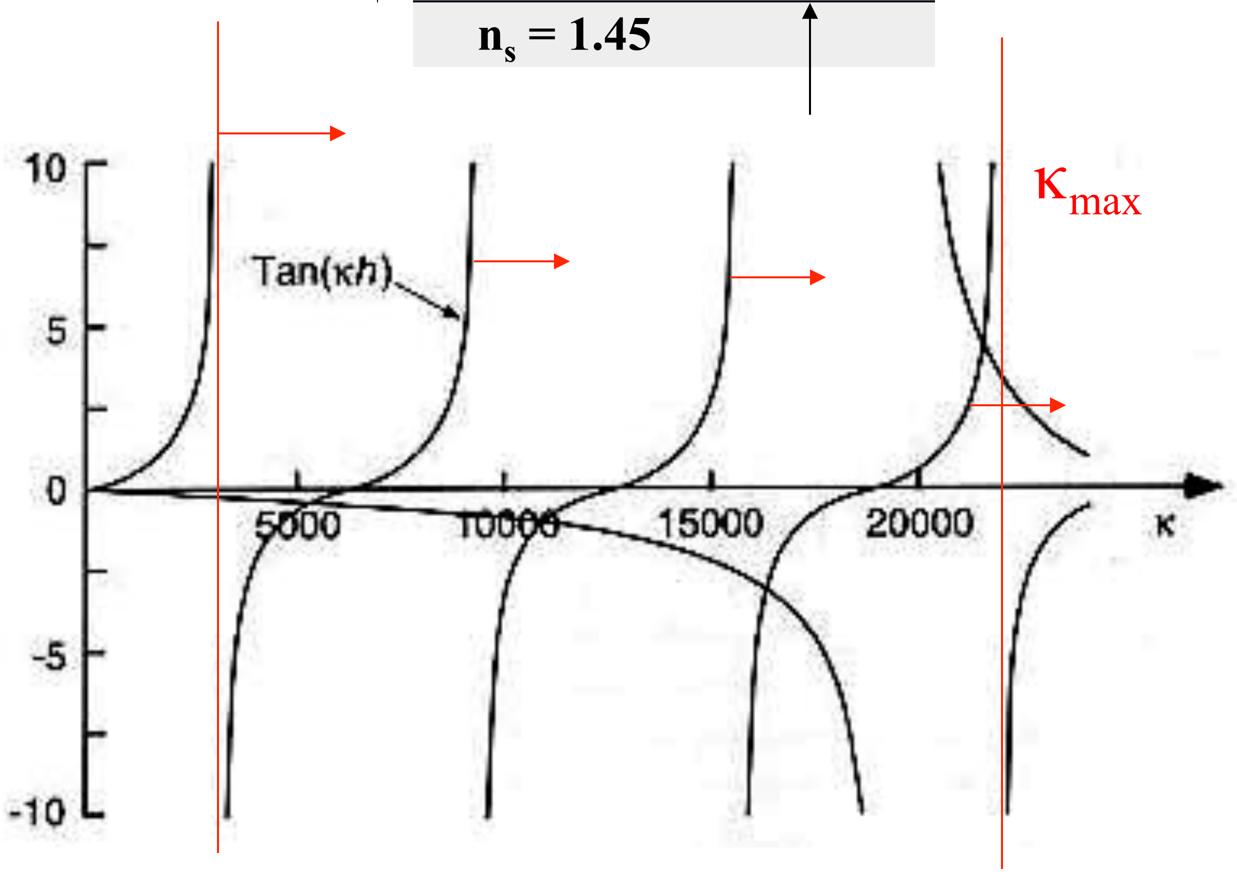
Slab WG Example





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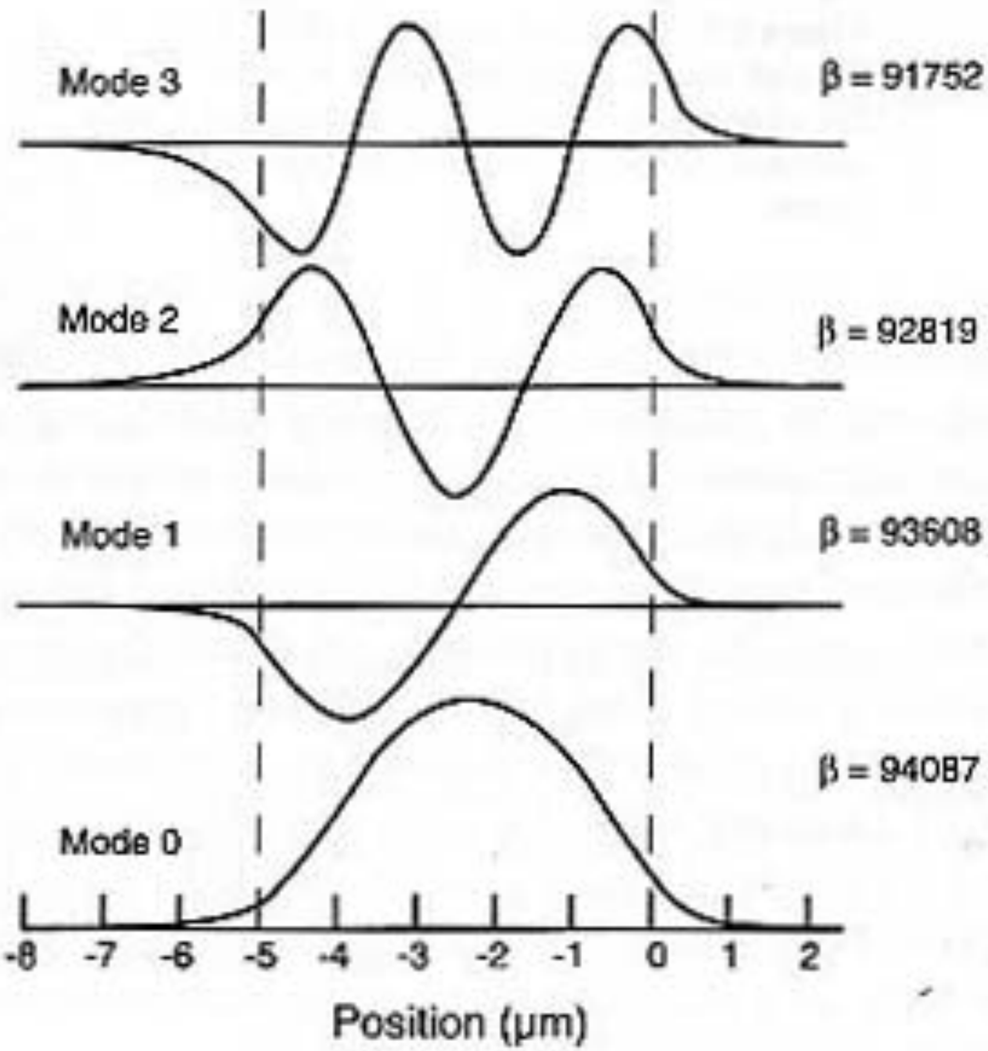
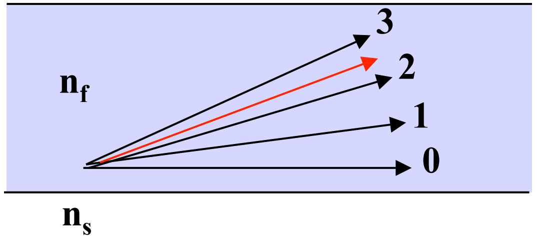
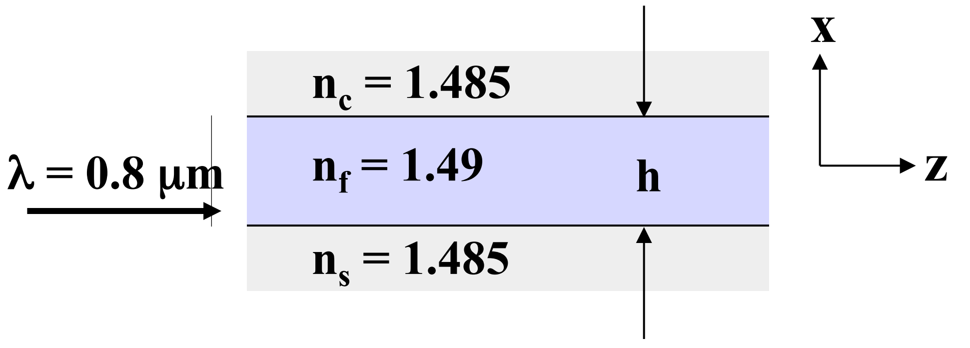
Mode Profiles

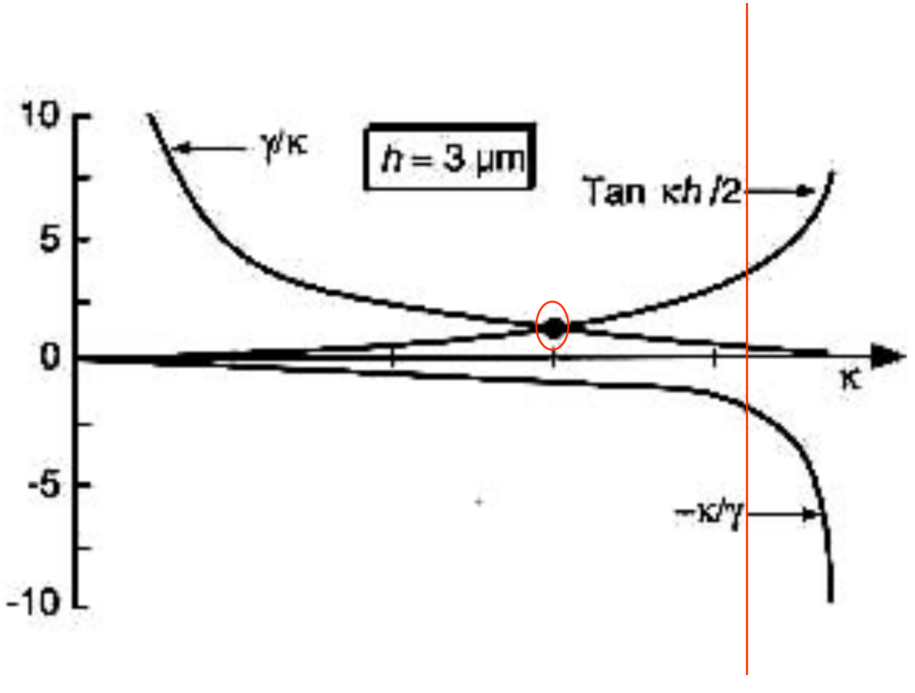


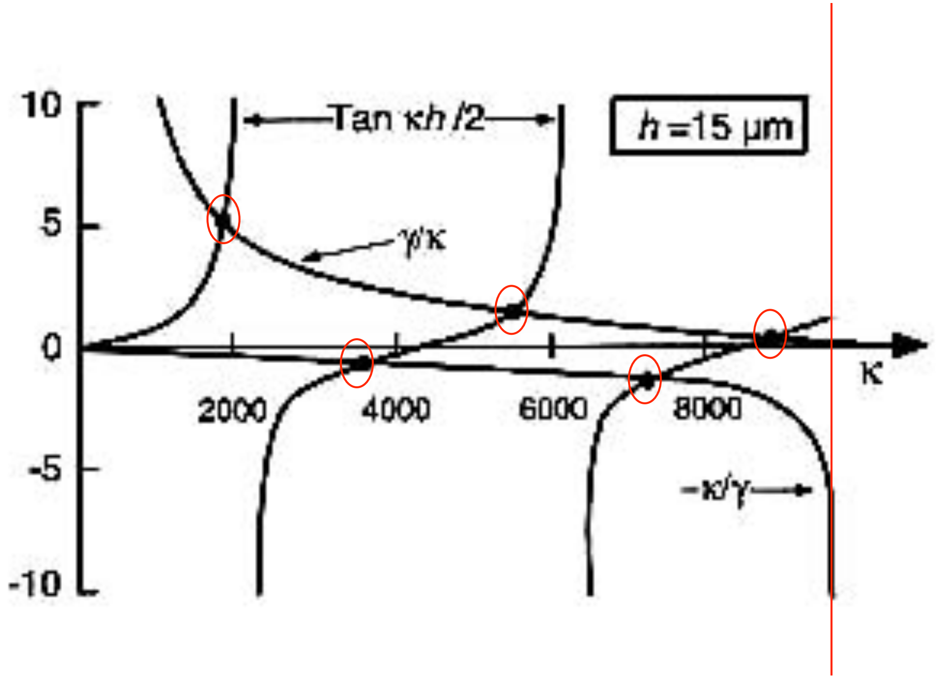


P_{core} / P_{total}	*Mode*
85.9 %	3
	2
	1
99.47 %	0

Symmetric Waveguide Example







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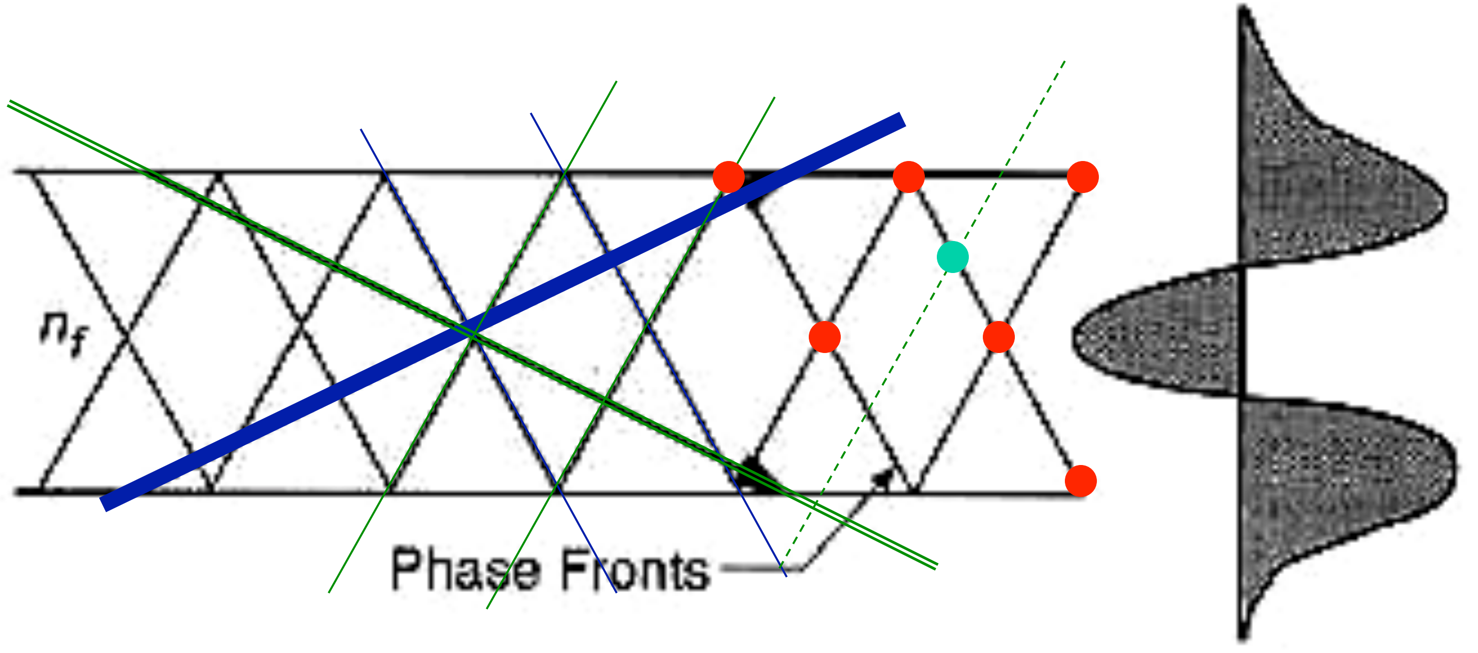
Intuitive Picture



Why is β Limited to Certain Values?

- The maximum value of β is $k_o n_f$

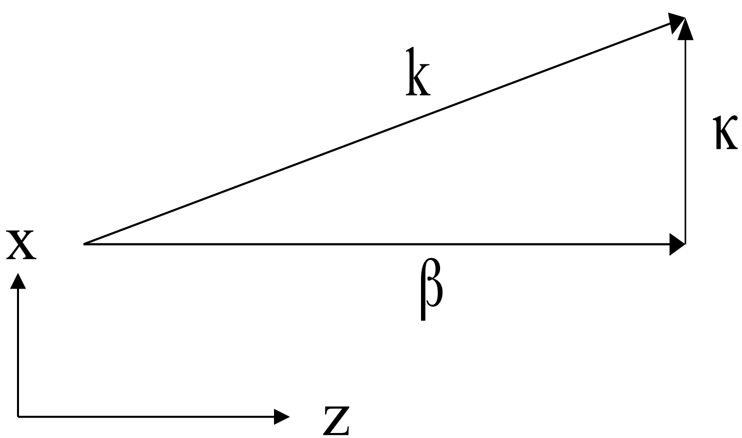
This corresponds to a ray parallel to z into the waveguide

- The minimum value of β is $k_o n_s$

This corresponds to a ray at the critical angle to the film-substrate interface

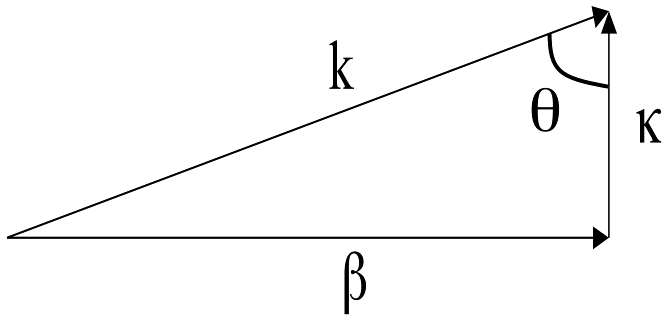
- There are only a *discrete number* of β in this range

A resonance condition must be satisfied if a mode is to be supported in the waveguide, only a discrete number of modes can satisfy this resonance condition

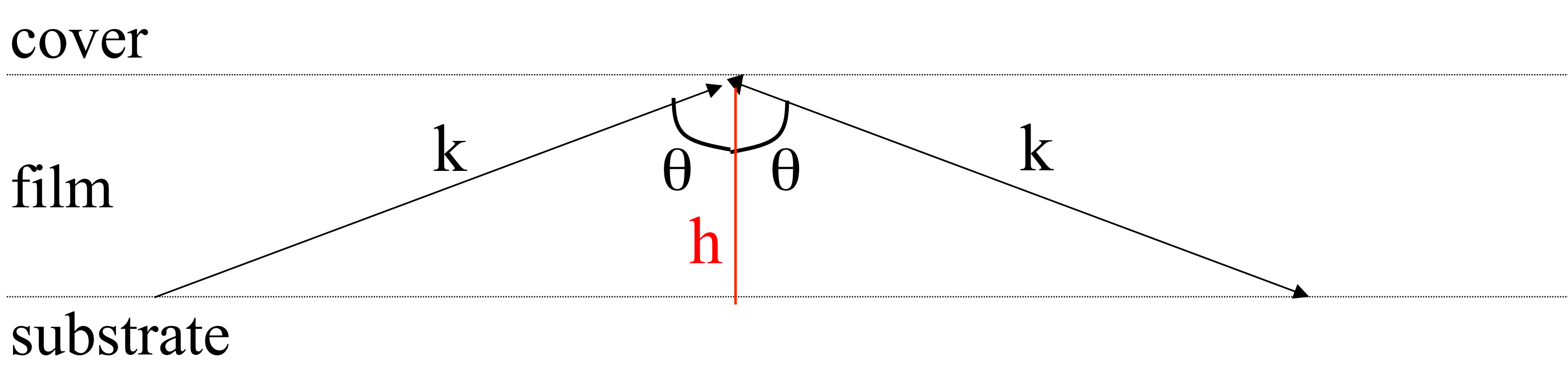


Transverse Resonance Condition

- *Confinement* in one dimension leads to
- *Standing waves* and confined *modes* in that dimension



Phase shift = k • d = 2 π ν,
ν is an integer



$$2\pi\nu = 2\, k_o\, n_f\, h \cos\theta - 2\Phi_c - 2\Phi_s$$

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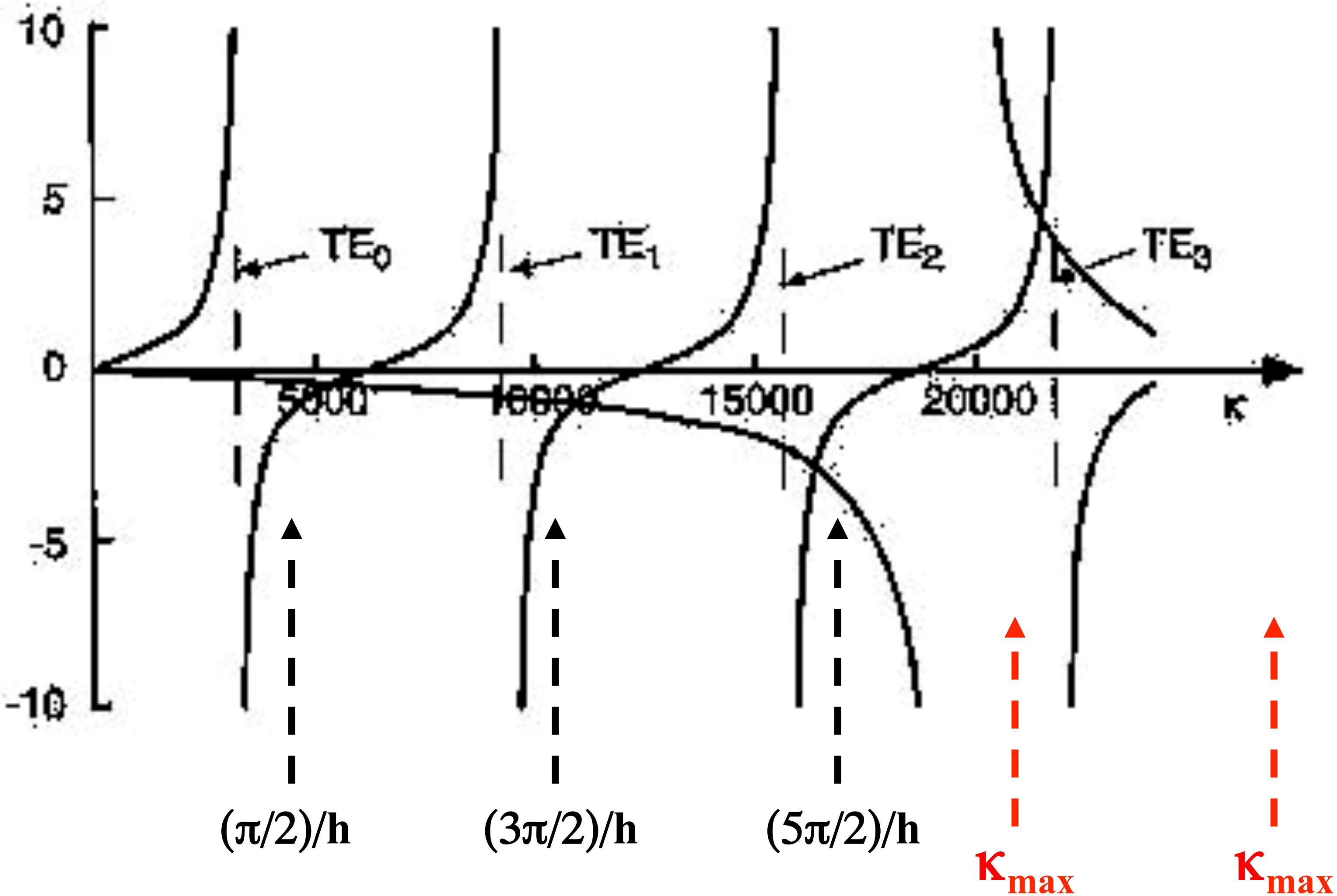
Number of Guided Modes



Pulse distortion and bandwidth are dependent on number of modes

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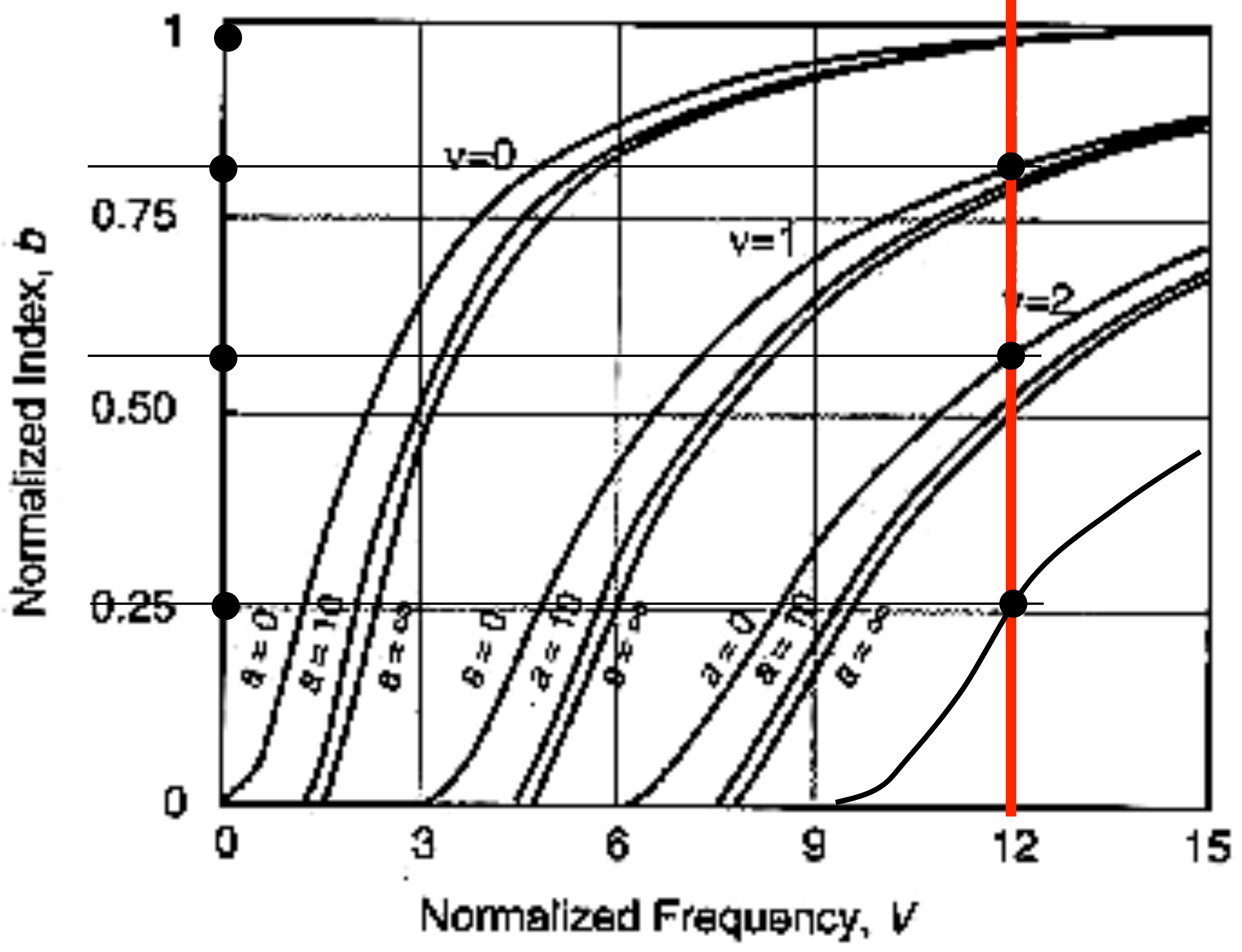
Number of Guided Modes

• Approximation below is best applied when there is a large number of modes:

$$m = \mathrm{Int}[\kappa_{max} h / \pi] = \mathrm{Int}[h\ k_o(n_f^2 - n_s^2)^{1/2} / \pi]$$

• Number of modes increase with thickness h, with index difference between film and cladding, and as light wavelength λ_o gets shorter.

Normalized Index: Slab Waveguide



Normalized Index: Slab Waveguide