

## CHAPTER 2 Dynamics, Chaos, and Prediction

*It makes me so happy. To be at the beginning again, knowing almost nothing. . . .  
The ordinary-sized stuff which is our lives, the things people write poetry  
about—clouds—daffodils—waterfalls. . . .these things are full of mystery, as  
mysterious to us as the heavens were to the Greeks. . . .It's the best possible  
time to be alive, when almost everything you thought you knew is wrong.  
—Tom Stoppard, Arcadia*

**D**ynamical systems theory (or *dynamics*) concerns the description and prediction of systems that exhibit complex *changing* behavior at the macroscopic level, emerging from the collective actions of many interacting components. The word *dynamic* means changing, and dynamical systems are systems that change over time in some way. Some examples of dynamical systems are

The solar system (the planets change position over time)  
The heart of a living creature (it beats in a periodic fashion rather than standing still)  
The brain of a living creature (neurons are continually firing, neurotransmitters are propelled from one neuron to another, synapse strengths are changing, and generally the whole system is in a continual state of flux)  
The stock market  
The world's population  
The global climate

Dynamical systems include these and most other systems that you probably can think of. Even rocks change over geological time. Dynamical systems theory describes in general terms the ways in which systems can change, what types of macroscopic behavior are possible, and what kinds of predictions about that behavior can be made.

Dynamical systems theory has recently been in vogue in popular science because of the fascinating results coming from one of its intellectual offspring, the study of chaos. However, it has a long history, starting, as many sciences did, with the Greek philosopher Aristotle.

### *Early Roots of Dynamical Systems Theory*

Aristotle was the author of one of the earliest recorded theories of motion, one that was accepted widely for over 1,500 years. His theory rested on two main principles, both of which turned out to be wrong. First, he believed that motion on Earth differs from motion in the heavens. He asserted that on Earth objects move in straight lines and only when something forces them to; when no forces are applied, an object comes to its natural resting state. In the heavens, however, planets and other celestial objects move continuously in perfect circles centered about the Earth. Second, Aristotle believed that earthly objects move in different ways depending on what they are made of.

For example, he believed that a rock will fall to Earth because it is mainly composed of the element *earth*, whereas smoke will rise because it is mostly composed of the element *air*. Likewise, heavier objects, presumably containing more earth, will fall faster than lighter objects.

Clearly Aristotle (like many theorists since) was not one to let experimental results get in the way of his theorizing. His scientific method was to let logic and common sense direct theory; the importance of testing the resulting theories by experiments is a more modern notion. The influence of Aristotle's ideas was strong and continued to hold sway over most of Western science until the sixteenth century—the time of Galileo.

Galileo was a pioneer of experimental, empirical science, along with his predecessor Copernicus and his contemporary Kepler. Copernicus established that the motion of the planets is centered not about the Earth but about the sun. (Galileo got into big trouble with the Catholic Church for promoting this view and was eventually forced to publicly renounce it; only in 1992 did the Church officially admit that Galileo had been unfairly persecuted.) In the early 1600s, Kepler discovered that the motion of the planets is not circular but rather elliptical, and he discovered laws describing this elliptical motion. Whereas Copernicus and Kepler focused their research on celestial motion, Galileo studied motion not only in the heavens but also here on Earth by experimenting with the objects one now finds in elementary physics courses: pendula, balls rolling down inclined planes, falling objects, light reflected by mirrors. Galileo did not have the sophisticated experimental devices we have today: he is said to have timed the swinging of a pendulum by counting his heartbeats and to have measured the effects of gravity by dropping objects off the leaning tower of Pisa. These now-classic experiments revolutionized ideas about motion. In particular, Galileo's studies directly contradicted Aristotle's long-held principles of motion. Against common sense, rest is *not* the natural state of objects; rather it takes *force* to stop a moving object. Heavy and light objects in a vacuum fall at the same rate. And perhaps most revolutionary of all, laws of motion on the Earth could explain some aspects of motions in the heavens. With Galileo, the scientific revolution, with experimental observations at its core, was definitively launched.

The most important person in the history of dynamics was Isaac Newton. Newton, who was born the year after Galileo died, can be said to have invented, on his own, the science of dynamics. Along the way he also had to invent calculus, the branch of mathematics that describes motion and change. Physicists call the general study of motion *mechanics*. This is a historical term dating from ancient Greece, reflecting the classical view that all motion could be explained in terms of the combined actions of simple “machines” (e.g., lever, pulley, wheel and axle). Newton's work is known today as *classical mechanics*. Mechanics is divided into two areas: kinematics, which describes how things move, and dynamics, which explains why things obey the laws of kinematics. For example, Kepler's laws are kinematic laws—they describe *how* the planets move (in ellipses with the sun at one focus)—but not *why* they move in this particular way. Newton's laws are the foundations of dynamics: they explain the motion of the planets, and everything else, in terms of the

basic notions of force and mass.

Newton's famous three laws are as follows:

1. Constant motion: Any object not subject to a force moves with unchanging speed.
2. Inertial mass: When an object is subject to a force, the resulting change in its motion is inversely proportional to its mass.
3. Equal and opposite forces: If object A exerts a force on object B, then object B must exert an equal and opposite force on object A.

One of Newton's greatest accomplishments was to realize that these laws applied not just to earthly objects but to those in the heavens as well. Galileo was the first to state the constant-motion law, but he believed it applied only to objects on Earth. Newton, however, understood that this law should apply to the planets as well, and realized that elliptical orbits, which exhibit a constantly *changing* direction of motion, require explanation in terms of a force, namely gravity. Newton's other major achievement was to state a universal law of gravity: the force of gravity between two objects is proportional to the product of their masses divided by the square of the distance between them. Newton's insight—now the backbone of modern science—was that this law applies everywhere in the universe, to falling apples as well as to planets. As he wrote: "nature is exceedingly simple and conformable to herself. Whatever reasoning holds for greater motions, should hold for lesser ones as well."

Newtonian mechanics produced a picture of a "clockwork universe," one that is wound up with the three laws and then runs its mechanical course. The mathematician Pierre Simon Laplace saw the implication of this clockwork view for prediction: in 1814 he asserted that, given Newton's laws and the current position and velocity of every particle in the universe, it was possible, in principle, to predict everything for all time. With the invention of electronic computers in the 1940s, the "in principle" might have seemed closer to "in practice."

### *Revised Views of Prediction*

However, two major discoveries of the twentieth century showed that Laplace's dream of complete prediction is not possible, even in principle. One discovery was Werner Heisenberg's 1927 "uncertainty principle" in quantum mechanics, which states that one cannot measure the exact values of the position and the momentum (mass times velocity) of a particle at the same time. The more certain one is about where a particle is located at a given time, the less one can know about its momentum, and vice versa. However, effects of Heisenberg's principle exist only in the quantum world of tiny particles, and most people viewed it as an interesting curiosity, but not one that would have much implication for prediction at a larger scale—predicting the weather, say. It was the understanding of *chaos* that eventually laid to rest the hope of perfect prediction of all complex systems, quantum or otherwise. The defining idea of chaos is that there are some systems—*chaotic* systems—in which even

minuscule uncertainties in measurements of initial position and momentum can result in huge errors in long-term predictions of these quantities. This is known as “sensitive dependence on initial conditions.”

In parts of the natural world such small uncertainties will not matter. If your initial measurements are fairly but not perfectly precise, your predictions will likewise be close to right if not exactly on target. For example, astronomers can predict eclipses almost perfectly in spite of even relatively large uncertainties in measuring the positions of planets. But sensitive dependence on initial conditions says that in chaotic systems, even the tiniest errors in your initial measurements will eventually produce huge errors in your prediction of the future motion of an object. In such systems (and hurricanes may well be an example) *any* error, no matter how small, will make long-term predictions vastly inaccurate.

This kind of behavior is counterintuitive; in fact, for a long time many scientists denied it was possible. However, chaos in this sense has been observed in cardiac disorders, turbulence in fluids, electronic circuits, dripping faucets, and many other seemingly unrelated phenomena. These days, the existence of chaotic systems is an accepted fact of science.

It is hard to pin down who first realized that such systems might exist. The possibility of sensitive dependence on initial conditions was proposed by a number of people long before quantum mechanics was invented. For example, the physicist James Clerk Maxwell hypothesized in 1873 that there are classes of phenomena affected by “influences whose physical magnitude is too small to be taken account of by a finite being, [but which] may produce results of the highest importance.”

Possibly the first clear example of a chaotic system was given in the late nineteenth century by the French mathematician Henri Poincaré. Poincaré was the founder of and probably the most influential contributor to the modern field of dynamical systems theory, which is a major outgrowth of Newton’s science of dynamics. Poincaré discovered sensitive dependence on initial conditions when attempting to solve a much simpler problem than predicting the motion of a hurricane. He more modestly tried to tackle the so-called **three body problem: to determine, using Newton’s laws, the long-term motions of three masses exerting gravitational forces on one another**. Newton solved the *two*-body problem, but the three-body problem turned out to be much harder. Poincaré tackled it in 1887 as part of a mathematics contest held in honor of the king of Sweden. The contest offered a prize of 2,500 Swedish crowns for a solution to the “many body” problem: predicting the future positions of arbitrarily many masses attracting one another under Newton’s laws. This problem was inspired by the question of whether or not the solar system is stable: will the planets remain in their current orbits, or will they wander from them? Poincaré started off by seeing whether he could solve it for merely three bodies.

He did not completely succeed—the problem was too hard. But his attempt was so impressive that he was awarded the prize anyway. Like Newton with calculus, Poincaré had to invent a new branch of mathematics, *algebraic*

*topology*, to even tackle the problem. Topology is an extended form of geometry, and it was in **looking at the geometric consequences of the three-body problem that he discovered the possibility of sensitive dependence on initial conditions**. He summed up his discovery as follows:

“If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon has been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomenon. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible. . . .”

In other words, *even if we know the laws of motion perfectly, two different sets of initial conditions (here, initial positions, masses, and velocities for objects), even if they differ in a minuscule way, can sometimes produce greatly different results in the subsequent motion of the system*. Poincaré found an example of this in the three-body problem.

It was not until the invention of the electronic computer that the scientific world began to see this phenomenon as significant. Poincaré, way ahead of his time, had guessed that sensitive dependence on initial conditions would stymie attempts at long-term weather prediction. **His early hunch gained some evidence when, in 1963, the meteorologist Edward Lorenz found that even simple computer models of weather phenomena were subject to sensitive dependence on initial conditions. Even with today’s modern, highly complex meteorological computer models, weather predictions are at best reasonably accurate only to about one week in the future.** It is not yet known whether this limit is due to fundamental chaos in the weather, or how much this limit can be extended by collecting more data and building even better models.