

This problem set is due on 2007 December 12, Wednesday 12pm.

1. A magnetic field  $\mathbf{B} = (B_0/3)(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$  exists at a point. For each of the following velocities of a test charge  $q$ , find the electric field  $\mathbf{E}$  at that point for which the acceleration experienced by the test charge is zero: (a)  $v_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z)$ ; (b)  $v_0(2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$ ; and (c)  $v_0$  along the line  $y = -z = 2x$ .
2. In a region of uniform electric and magnetic fields  $\mathbf{E} = E_0\mathbf{a}_y$  and  $\mathbf{B} = B_0\mathbf{a}_z$ , respectively, a test charge  $q$  of mass  $m$  moves in the manner

$$\begin{aligned}x &= \frac{E_0}{\omega_c B_0}(\omega_c t - \sin \omega_c t) \\y &= \frac{E_0}{\omega_c B_0}(1 - \cos \omega_c t) \\z &= 0\end{aligned}$$

where  $\omega_c = qB_0/m$ . Find the forces acting on the test charge for the following values of  $t$ : (a)  $t = 0$ ; (b)  $t = \pi/2\omega_c$ ; and (c)  $t = \pi/\omega_c$ .

3. Consider a test charge moving with constant velocity  $\mathbf{v} = v_0(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$  in a region of uniform electric field of intensity  $\mathbf{E} = E_0(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$  and a uniform magnetic field of flux density  $\mathbf{B} = B_{x0}\mathbf{a}_x + B_{y0}\mathbf{a}_y + B_{z0}\mathbf{a}_z$ . Is this information sufficient to find uniquely  $B_{x0}$ ,  $B_{y0}$  and  $B_{z0}$ ? If not, given the  $\mathbf{v}$  is perpendicular to  $\mathbf{B}$ , find  $B_{x0}$ ,  $B_{y0}$ , and  $B_{z0}$  in terms of  $E_0$  and  $v_0$ .
4. Three forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$  and  $\mathbf{F}_3$  experienced by a test charge  $q$  at a point in a region of electric and magnetic fields for three different velocities of the test charge are given as follows:

$$\begin{aligned}\mathbf{F}_1 &= \mathbf{0} & \text{for } \mathbf{v}_1 &= v_0\mathbf{a}_x \\ \mathbf{F}_2 &= \mathbf{0} & \text{for } \mathbf{v}_2 &= v_0\mathbf{a}_y \\ \mathbf{F}_3 &= qE_0\mathbf{a}_z & \text{for } \mathbf{v}_3 &= v_0(\mathbf{a}_x + 2\mathbf{a}_y)\end{aligned}$$

Find the forces  $\mathbf{F}_4$ ,  $\mathbf{F}_5$ , and  $\mathbf{F}_6$  experienced by the test charge at that point for three other velocities: (a)  $\mathbf{v}_4 = \mathbf{0}$ ; (b)  $\mathbf{v}_5 = v_0(\mathbf{a}_x + \mathbf{a}_y)$ ; and (c)  $\mathbf{v}_6 = (v_0/4)(3\mathbf{a}_x + \mathbf{a}_y)$ .

5. Uniform electric and magnetic fields exist in a region of space. A test charge  $q$  released with an initial velocity  $\mathbf{v}_1$  or  $\mathbf{v}_2$  moves with constant velocity equal to the initial value. Show that the test charge moves with constant velocity equal to the initial value when released with an initial velocity  $(m\mathbf{v}_1 + n\mathbf{v}_2)/(m + n)$  for any nonzero  $(m + n)$ .